Quantum Mechanics

Plank's Quantum Theory of Radiation

According to plank's Quantum theory a heat body emits radiations in all directions depending on the size, surface & temperature of the body.

When these radiations are incident on a surface a part gets reflected, a part gets absorbed and another part gets transmitted.

$r + a + t = 100\%$

$r \rightarrow$ reflected part
$a \rightarrow$ absorbed part
$t \rightarrow$ transmitted part

Perfect black body

A perfect black body is the one which absorbs all the radiations coming in to it. There will be neither the reflection nor transmission of radiations.

ie, $a = 100\%$ for black body.

But Generally $r + a + t = 100\%$.

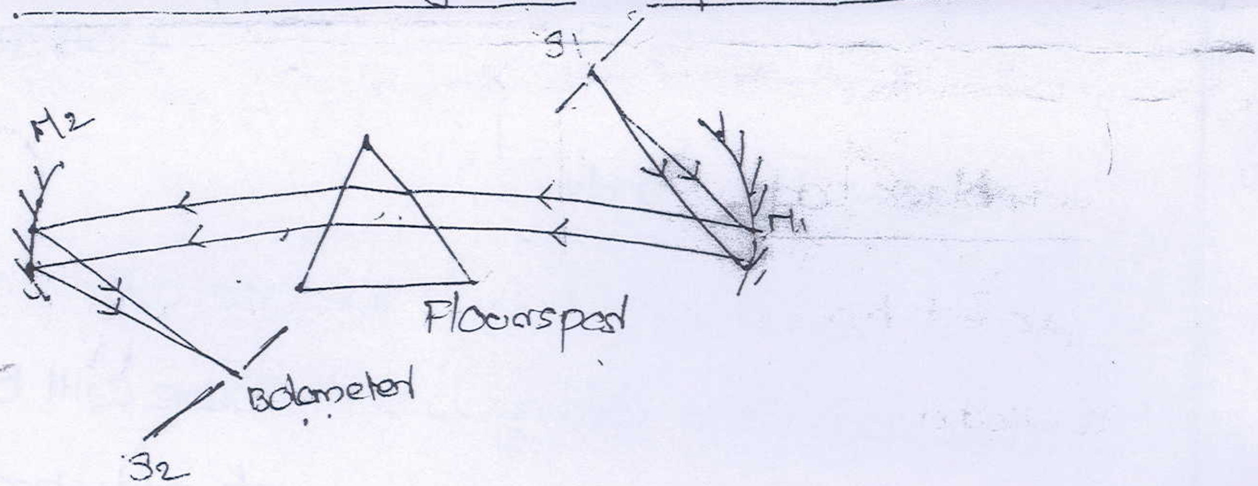
When black body is heated it starts emitting the absorbed radiations.

Ferary's black body

It is a double walled hollow sphere. It con
a fine hole 'O' at one end and a projection 'p' at
other end. The inner walls are coated with lamp bl
The projection 'p' prevents direct reflections.

A light ray entering through 'O' undergoes complete reflections i.e, ~~total~~ multiple reflections and finally completely absorbed by the body.

Distribution of Energy in the spectrum of a black body



Distribution of Energy of radiations was first analysed Lummer and Pringsheim. The radiations from heated carbon tube passess through slit S_1, reflected by mir M_1 and dispersed through floorspad prism and finall focussed to a bolometer which measures the intensity.

By rotating the prism intensities corresponding different wavelength are determined ....

A graph is plotted with wavelength along X-axis and intensity along Y-axis.

Results of experiment :-

① Energy distribution is not uniform over the spectrum

② At a particular temperature the intensity first increases and reaches a maximum value λ_m then decreases.

③ increase in temperature produces increase in Energy

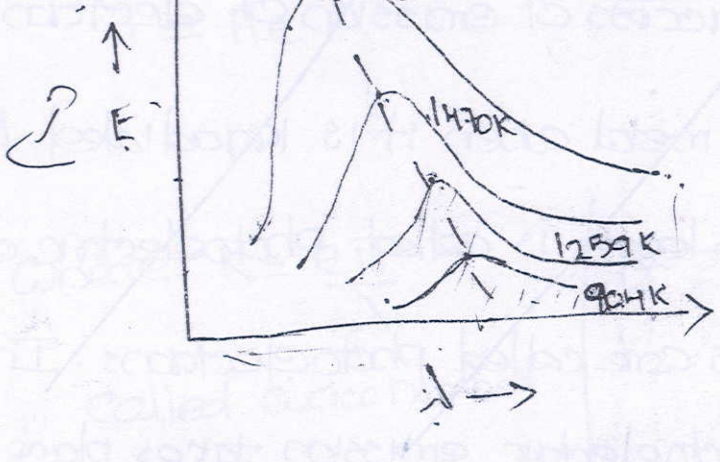
④ As temperature increases λ_m shifted towards the lower wavelength side.

$$\lambda_m \propto \frac{1}{T}$$

⑤ The area b/w curve and wavelength axis gives the total energy emitted per unit area per unit sec

ie, Energy (area) is directly proportional to Fourth Power of it's absolute temperature

$E \propto T^4$ — stefan's law.



$E \propto T^4$ — stefans law

Plank's hypothesis of Quantum theory of ra

classical theory fails to explain Energy distribu a black body. It plank explained it with Quantum t

plank's hypothesis are:

(1) A blackbody radiation chamber contains Bolometers, radiators and simple harmonic oscillators. They vibrate with all possible frequencies.

(2) Frequency of emitted radiation will be same as of Vibration of oscillator.

(3) An oscillator cannot continuously emits radiation. It emits radiations in integral multiple of quantum of energy $h\nu$.

(4) An oscillator can absorb or emit radiations as packets of energy $nh\nu$.

Photoelectric effect

The process of emission of electrons from the surface of a metal when it is irradiated by light of particular wave length is called photoelectric effect. The emitted photons are called photoelectrons. In the case o alkali metal photoelectric emission takes place due to

Schrodinger's wave equation for a free Particle. Derive Schrodingers time dependent equation? [Essay Question] Ramsum M Dsc

In the case of schrodinger's time dependent equation the final equation will contain time related terms. Consider a debroglie wave. It is a particle associated with a wave or wave associated with a particle. The wave is assumed to be moving in x-direction with a wavelength $\lambda = \frac{h}{p}$, the debroglie wave length.

The wave equation is given by

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\frac{d^2\psi}{dt^2} \quad \text{——— (1)}$$

The solution is given by

$$\psi = Ae^{i(kx-\omega t)} \quad \text{——— (2)}$$

Where $K = \frac{2\pi}{\lambda}$ called wave number | $\omega = 2\pi\nu$ called angular frequency.

$$E = h\nu$$

$$E = h\nu \cdot \frac{2\pi}{2\pi}$$

$$E = \frac{h}{2\pi} \cdot 2\pi\nu$$

$$E = \hbar\omega$$

$$\omega = E/\hbar \quad \text{—— (3)}$$

$$\lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda}$$

$$p = \frac{h}{\lambda} \cdot \frac{2\pi}{2\pi}$$

$$p = \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda}$$

$$p = \hbar k$$

$$k = p/\hbar \quad \text{—— (4)}$$

Substitute the value of ω & k from eq (3) (4) to eq (2).

$$\psi = Ae^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)}$$

ie,

$$\psi = Ae^{i/\hbar (px - Et)} \quad \text{—— (5)}$$

Now applying Energy & momentum operation in

Differentiating ~~twice~~ eq (5) w.r.t x.

$$\frac{d\psi}{dx} = Ae^{i/\hbar(px-Et)} \cdot \frac{i}{\hbar} p$$

$$\frac{d\psi}{dx} = \psi \frac{i}{\hbar} p$$

~~cancelling ψ on both sides~~.

$$\frac{d}{dx} = \frac{i}{\hbar} p \quad (7)$$

$$\frac{P}{\hbar} \quad P = \frac{\hbar}{i}\frac{d}{dx}$$

$$P = \frac{\hbar}{i}\frac{d}{dx}\frac{i}{i}$$

$$\boxed{\hat{p} = -i\hbar\frac{d}{dx}} \quad (6)$$

Differentiating eq (5) w.r.t "t"

$$\frac{d\psi}{dt} = Ae^{\frac{i}{\hbar}(px-Et)} \cdot -E\frac{i}{\hbar}$$

$$\frac{d\psi}{dt} = -\psi E\frac{i}{\hbar}$$

~~Cancelling ψ on both sides.~~

$$\frac{d}{dt} = -E\frac{i}{\hbar}$$

$$E = \frac{\hbar}{i}\frac{d}{dt}$$

$$E = -\frac{\hbar}{i}\frac{d}{dt}\cdot\frac{i}{i}$$

$$\boxed{\hat{E} = i\hbar\frac{d}{dt}} \quad (7)$$

1 15 20 21 28 31 35 38 41

Total Energy = K.E + P.E

$$E = \frac{1}{2}mv^2 + V$$

$$E\psi = \frac{p^2\psi}{2m} + V\psi$$

$$= \frac{1}{2}mv^2 \cdot \frac{m}{m} = \frac{m^2v^2}{2m} = \frac{p^2}{2m}.$$

$$i\hbar\frac{d\psi}{dt} = \frac{\left(-i\hbar\frac{d}{dx}\right)^2\psi}{2m} + V\psi$$

$$\boxed{i\hbar\frac{d\psi}{dt} = \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi}$$

$$\boxed{i\hbar\frac{d\psi}{dt} = \frac{-\hbar^2}{2m}\nabla^2\psi + V\psi}$$

The equation represents schrodingers time dependent equation.

Schrodingers time independent equation

Potential in schrodinger equation or steady state equation.

In 1924 Louis de-Broglie suggested that particles are associated with waves. These waves are called matter waves. To explain the characteristics of matter waves schrödinger introduced the idea of wave function. It is represented by ψ. It is a function of space and time. It gives the probability of finding the particle at a particular point at a particular time. $\psi(x,y,z,t)$ representing matter waves can completely describe the behaviour of particle associated with wave. The probability of finding the particle in a volume $dx\,dy\,dz$ is given by $|\psi|^2 dx\,dy\,dz$. Here $|\psi|^2$ is called probability density. Since maximum probability is one, $|\psi|^2 dx\,dy\,dz = 1$. A wave function satisfying the above condition is called Normalised wave function.

(1) According to Louis de Broglie an electron of mass 'm' moving with velocity v_g behaves as a wave of wavelength $\lambda = \frac{h}{mv}$. Assuming that the wave is moving in x-direction. The differential equation is $\frac{d^2\psi}{dt^2} = v^2 \frac{d^2\psi}{dx^2}$ (A)

The solution to this equation in cartesian co-ordinate

System is

$$\psi = \psi_0 \sin 2\pi \nu t \quad \text{---} (1)$$

$$\frac{d^2\psi}{dx^2} = -4\pi^2\nu^2\psi \quad \text{---} (2)$$

Substituting this in differential eqn (A)

$$-4\pi^2\nu^2\psi = v^2 \frac{d^2\psi}{dx^2}$$

$$\frac{d^2\psi}{dx^2} = \frac{-4\pi^2\nu^2\psi}{v^2}$$

$\therefore$ But $v = \nu\lambda$

$$\frac{d^2\psi}{dx^2} = \frac{-4\pi^2\psi}{\lambda^2}$$

$\therefore$ But for matter waves $\lambda = \frac{h}{mv}$

$$\frac{d^2\psi}{dx^2} = \frac{-4\pi^2 m^2 v^2 \psi}{h^2} \quad \text{---} (3)$$

But total energy $E = K.E + P.E$

$$= \frac{1}{2}mv^2 + V$$

$$\frac{1}{2}mv^2 = E - V, \quad mv^2 = 2(E-V)$$

$$m^2v^2 = 2m(E-V) \quad \text{---} (4)$$

Substituting (4) in (3)

$$\frac{d^2\psi}{dx^2} = \frac{-8\pi^2 m(E-V)\psi}{h^2}$$

ie, $\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m(E-V)}{h^2}\psi = 0$

But $\hbar = h/2\pi$

substituting

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0} \longrightarrow (5)$$

This gives the schrodinger's one dimensional wave equation for matter waves. If we consider a three dimensional motion then the schrodinger's equation will be

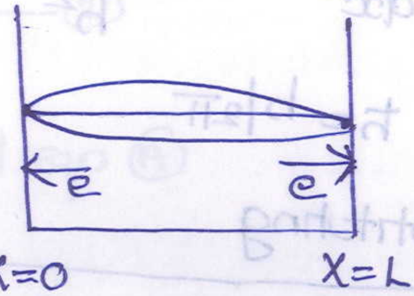
$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{—— (6)}$$

It is the steady state form and has no dependence on time.

some of the application of schrodinger's wave equation is

1. particle in a box
2. particle in a potential well.
3. Tunnel effect
4. Harmonic oscillator
5. Hydrogen atom.

(2) Write or Derive the Schrodinger's wave eq for a particle in a box?



Consider a particle of mass 'm' moving inside a one dimensional box of length 'L' along the x-direction between X=0 and X=L. So the particle will be bouncing back and forth between the wall of the box. Let us assume that particle is loosing any energy when it collides with walls of box. Let us also assume the Potential energy inside box is zero. So the schrodinger's wave equation inside the box is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0$$

ie, $\frac{d^2\psi}{dx^2} + k^2\psi = 0$ —— (1)

where $k^2 = \frac{8\pi^2 m}{h^2}(E-V)$

V=0, Since Potential Energy is zero.

So solution to eqn (1) is

$\psi = A\sin kx + B\cos kx$ —— (2)

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At $x=0$ & $\psi=0$

Then $B=0$

Then eqn (2) will be

$$\psi = A\sin kx \quad \text{—(3)}$$

At $x=L$ & $\psi=0$

Then eqn (3) will be

$$A\sin kL = 0$$

$$kL = n\pi$$

$$k = \frac{n\pi}{L}, \quad k^2 = \frac{n^2\pi^2}{L^2} \quad \text{—(4)}$$

Since $k^2 = \frac{8\pi^2 mE}{h^2}$ we have

$$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{L^2}$$

$$\boxed{E_n = \frac{n^2h^2}{8mL^2}} \quad \text{—(5)}$$

(3) Eigen wave functions

(Write at the end of Particle in a box essay question.)

Here 'n' is called Quantum Number which can take values 1, 2, 3 etc.

So $\psi_n = A\sin kx$ [from eqn (3)]

ie, $\psi_n = A\sin \frac{n\pi x}{L}$ [from (4)]

For each value of 'n' there is an energy and a corresponding wave function given by ψ_n. Values are called the energy levels of particle in a Potential well. The particles cannot have zero energy since $n=1,2,3$ etc. so energy levels are Quantised.

considering the condition for Normalisation

$$\int \psi^* \psi \, dx = 1$$

$$\int_0^L |\psi|^2 dx = 1 \quad \text{——} (1)$$

$$\int_0^L A^2 \sin^2 \frac{n\pi x}{L} \, dx = 1$$

$[2\sin^2\theta = 1 - \cos 2\theta]$

$$\int_0^L \frac{A^2}{2} \left[1 - \cos \frac{2\pi n x}{L}\right] dx = 1$$

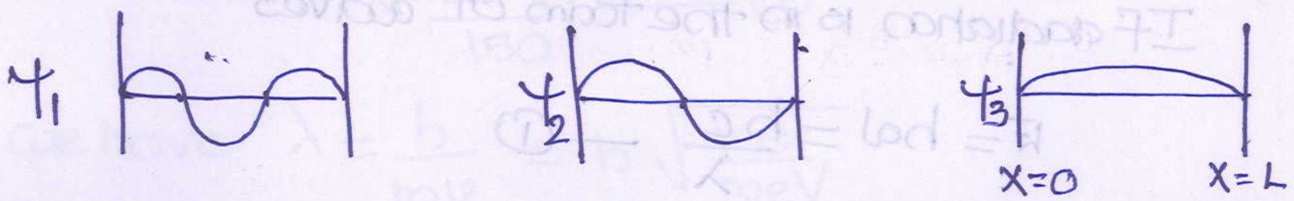
$$\frac{A^2}{2} \left[x - \frac{L}{2\pi n} \sin \frac{2\pi n x}{L}\right]_0^L = 1$$

$$\frac{A^2}{2} [L - 0] = 1$$

$$A^2 = \frac{2}{L} \quad \text{ie, } A = \sqrt{2/L} \quad \text{——} (2)$$

$$\text{ie, } \boxed{\psi_n = \sqrt{2/L} \sin \frac{n\pi x}{L}} \quad [\text{from } \psi_n = A \sin kx]$$

⇒ This is the normalised wave function for the particle.
The normalised wave function for n=1, n=2, n=3 is plotted below.



This characteristic value of the energy system is known as eigen value and the corresponding value of 'ψ' is called eigen function

(4) Matter waves or de-Broglie waves

Phenomena like interference, diffraction and polarisation shows that radiation is in the form of waves. But phenomena like compton effect & photoelectric effect shows that radiation is in the form of particles. So Louis de-Broglie a French scientist suggested that particles are ~~associated~~ associated with waves. These waves are called Matter waves or pilot waves or de-Broglie waves. In other words a wave associated with a particle in motion is called matter wave.

(5) Expression for De Broglie wave length ?

Expression for De Broglie wave length of for an elec

Ans:-

If radiation is in the form of waves

$$E = h\nu = \frac{hc}{\lambda} \quad \text{——} (1)$$

If radiation is in the form of particles

$$E = mc^2 \quad \text{——} (2)$$

comparing (1) & (2)

$$\frac{hc}{\lambda} = mc^2$$

$$\text{or } \lambda = \frac{h}{mc} \qquad c = \text{velocity of light}$$

$$\boxed{\lambda = \frac{h}{mv}} \text{ or } \boxed{\lambda = \frac{h}{p}} \quad \text{——} (3)$$

Eqn (3) is called de Broglie wavelength.

If the electron acquires a velocity v due to the voltage V. Then workdone is

$$eV = \frac{1}{2} m v^2$$

$$m v^2 = 2eV$$

$$m^2 v^2 = 2meV \quad \text{enu}$$

$$\left[emu = \frac{1}{300} esu \right]$$

$$m^2 v^2 = \frac{2meV}{300} \text{ esu}$$

$$\cancel{mv = \sqrt{2meV}}$$

$$mv = \sqrt{\frac{meV}{150}} \quad \text{--- (4)}$$

We have $\lambda = \frac{h}{mv} = h\sqrt{\frac{150}{meV}}$

Substituting the known Values of h, m, e & V we get

$$\lambda = \sqrt{\frac{150}{V}} \times 10^{-10} m$$

$$\boxed{\lambda = \sqrt{\frac{150}{V}} \, A^0} \quad \text{--- (5)}$$

Therefore the wavelength of matter waves is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} = \sqrt{\frac{150}{V}} \, A^0$$

(6) Heisenberg's uncertainity principle
— position & momentum

It ~~is~~ states that it is impossible to determine the position and momentum of a particle simultaneously and precisely.

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x \cdot \Delta p \geq \hbar/2 \quad \text{where } \hbar = h/2\pi$$

(6) Heisenberg's uncertainity principle – Energy & time

It states that it is impossible to determ[ine] the Energy and time of a particle simultaneously precisely.

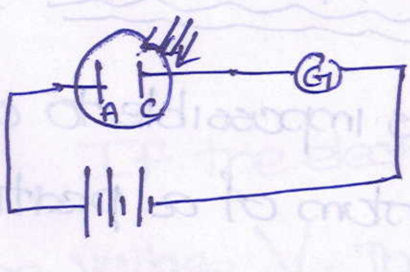
$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$

$$\Delta E \cdot \Delta t \geq \hbar/2$$

(7) Explain photoelectric effect? Derive photo electric equation [Learn More from Text – Essay Qn]

The phenomenon of emission of electrons when light is incident on certain materials is called photoelectric effect. The emitted electrons are called photoelectrons.

Experimental Arrangement :-



The experimental arrangement consists of two plates A & C placed in an evacuated tube. A galvanometer G and battery E are connected in series. When UV light is incident on the plate 'C' a current flows through e current.

Laws of photo electric emission :-

① Then emission of electron takes place only when the incident light has a minimum frequency ν_0. This frequency is called Threshold frequency.

② The intensity of incident light is directly proportional to photo current produced. when intensity is high, large no: of photo electrons are produced so photo current will be high.

③. The frequency of the incident light is related to the kinetic energy of the emitted photoelectrons. If frequency is high, the photoelectrons has an high K.E.

④ photoelectric emission is an instantaneous process. There is no time delay between hitting of light and emission of electron.

Failure of Em theory :-

① calculations showed ~~that~~ a delay of 500 day between hitting of ~~electron~~ light & emission of electron in the case of sodium. But experimentally there will be no time delay.

(2) According to classical theory intensity is ... to kinetic energy of electron and velocity ... no dependence with kinetic energy. But these are wrong.

Einsteins photoelectric equation

According to Einstein a part of energy of incident light is firstly utilised to pull out the electron from the surface of metal and remaining part is utilised for imparting kinetic energy.

Then $h\nu = \phi + \frac{1}{2}mv^2$ —— (1)

ϕ is called work function and is given by $\phi = h\nu_0$

$$h\nu = h\nu_0 + \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 = h\nu - h\nu_0$$

$$\boxed{\frac{1}{2}mv^2_{max} = h(\nu - \nu_0)} \quad \text{—— (2)}$$

V_{max} is the maximum kinetic energy attained by electron.

Derive schrodingers time dependent equation for a free particle

Please see pages from 355 to 357 of Text Book.

... tes

... ht

... is incident on the plate ...

Most important Frequently Asked problem Questions

1) Newton's ring $\lambda = \frac{D_{n+k}^2 - D_n^2}{4kR}$, $D_{n+k}^2 - D_n^2 = \frac{4Rk\lambda}{\mu}$ (liquid) [Page NO: 33 of Text]

Imp 2) Airwedge $d = \frac{\lambda l}{2\beta}$ angle of airwedge $\theta = \frac{\lambda}{2\mu\beta}$, $\theta = \frac{\lambda}{2\beta}$ [Page NO: 37 of Text]

Imp 3) condition for brightness & darkness eg: $2\mu t \cos r = n\lambda$
$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

Imp 4) grating equation $\sin\theta = Nn\lambda$ [Page NO: 67 of Text]

Imp 5) Thickness of Quarterwave plate $- t = \frac{\lambda}{4(\mu_o - \mu_e)}$
& Half wave plate $- t = \frac{\lambda}{2(\mu_o - \mu_e)}$

Imp 6) specific rotatory power $S = \frac{10\theta}{lc}$ [Page NO: 109 & 110]

7) Miller indices Determination.

8) Lattice constant $a = \left[\frac{Zm}{N_A \rho}\right]^{1/3}$ Page NO: 147.

Imp 9) $d_{hkl} = \frac{a}{\sqrt{h^2+k^2+l^2}}$ [Page NO: 142]

10) $n\lambda = 2d\sin\theta$ Bragg's law page 158, 159.

V. Imp 11) calculation of NA, θ_c, θ_a, Δ, V-number [Page 239-241]

V. Imp 12) calculation of H_c, I_c, J_c form superconductivity [Page 329-331]

$H_c = H_0\left(1 - \frac{T^2}{T_c^2}\right)$ $I_c = 2\pi r H_c$
$J_c = I_c / A$

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(13) De broglie wavelength $\lambda = \frac{h}{mv}$

De broglie wavelength for an electron $\lambda = \sqrt{\frac{150}{V}}\ A^0$

(14) calculation of fundamental frequency $\nu = \frac{1}{2l}\sqrt{Y/\rho}$

[Pages from 397 - 399]

(18) calculation of Reverberation time $T = 0.\frac{163V}{A}$

(15) calculation of Intensity in decibels.

$$L = 10 \log_{10}\left(\frac{I}{I_0}\right)$$